Wave - Mechanics

The Wave Mechanical Concept Comes in figure when L. de Broglie (1924) supported the Dual nature of electron i.e. Particle & Wave Nature. The mathematics of Wave Mechanics was developed by Schrodinger in 1926. When the exactness of Classical Mechanics has been replaced by Probability.

Now we can say the development of Wave Mecha. is based on the following concept.

1. L. de- Broglie's idea of dual nature of Matter.
2. Heisenberg's uncertainty Principle.
3. Schrodinger's Wave Equation.

1. Dual Nature of an Electron: - We ~~have~~ already discussed in Bhor's theory that electron posses Particle nature and revolving around nucleous in a Circular Orbits. But de Broglie Pointed out in 1924 that the electron like light behaves both as a material Particle and as a Wave. i.e electron has dual character.

de - Broglie's Equation: — de- Broglie derived an expression for Calculating the Wave length λ of the (Wave associated with electron) electron wave. If m be the mass of the electron and it is revolving around nucleous with Velocity c. The Velocity and wave length are related with following relation.

$$\lambda = \frac{h}{mc} \quad \text{——— (1)}$$

Where 'h' is planck's Constant $[h = 6.62 \times 10^{-34}$ jule - sec$]$ and mc is the momentum of the moving Particle.

The de- Broglie Relationship may be written as,

$$mc = \frac{h}{\lambda} \quad \text{or} \quad mc \propto \frac{1}{\lambda} \quad \text{— (2)}$$

This equation (2) is another form of de - Broglie's Equation. and it may be stated as follows.

" The momentum of a moving Particle is inversely proportional to the wave length of the wave associated with it.

Proof of the Lewis de-Broglie Equation:—

Let us consider the case of Photon. If we consider it to be a wave of frequency ν, then its energy is given by Plank Quantum theory:—

$$E = h\nu \quad \text{——(i)}$$

$$\left(\because \nu = \frac{c}{\lambda} \;\therefore\; E = \frac{hc}{\lambda}\right)$$

If we consider it as "a Particle of mass 'm' moving with velocity 'c'. Then its energy is given by Einstein mass Energy relation.

$$E = mc^2 \quad \text{——(ii)}$$

From Equation (i) and (ii)

$$mc^2 = h\nu$$

$$mc^2 = h\frac{c}{\lambda}$$

$$\text{Or } mc = \frac{h}{\lambda}$$

$$\text{or } \lambda = \frac{h}{mc} \quad \text{——(iii)}$$

$$\text{or } \lambda = \frac{h}{P} \quad (\text{where } P \text{ is momentum})$$

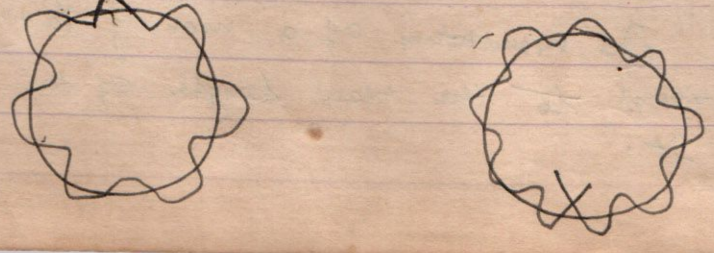
It is required proof of de-Broglie's Equation.

Similarity between de-Broglie's Wave-character of the electron and Bohr's Theory ✓

1. Quantisation of Angular Momentum:—

According to de-Broglie the electron is not a Solid Particle revolving round the Nucleous in a Circular Orbit, but it is a standing wave extended round the Nucleous in a Circular Orbit.

If r be the Radius of the Circular Orbit then its Circumference of this orbit is equal to $2\pi r$.



Now if λ is wave-length an is be the Total number (which is a whole number like 1,2,3 ....) of the wave lengths associated with the electron wave extending round the Nucleus. For the wave to remain continually in Phase, the circumference of the orbit should be integral multiple of wave length λ.

i.e. $2\pi r = n\lambda$ ——— (ii)

From equation (i) we know, $\lambda = \frac{h}{mc}$

Now substituting the value λ in eqn (ii)

$$2\pi r = n \cdot \frac{h}{mc}$$

$$\text{or } mcr = n \cdot \frac{h}{2\pi} \quad \left[mvr = n \cdot \frac{h}{2\pi} \right] \text{ — (iii)}$$

It is same as Bohr's Second Postulate.
Electron can move only in such orbits for which the angular momentum must be an integral multiple of $\frac{h}{2\pi}$.

Thus de-Broglee relation provides a theoretical basis for the Bohr's Second Postulates.

Experimental Verification of de-Broglie's Concept.
Experimental Verification of de-Broglie's Concept —
Or Verification of wave Nature of Electron:—

The de-Broglee Postulate received direct experimental verification in 1927. By Davision and Germe. Similar experiments were carried out independently by G.P. Thomson and later Stern showed that beams of heavier Particles (H_2, He, etc) showed diffraction patterns when reflected from the surface of crystals. Thus, de-Broglie expression for the wave lengths of these matter wave was confirmed with high accuracy.

In 1927 they succeeded in diffracting a beam of electron by means of a nickel surface the pattern of electron diffraction was found to be similar to that of X-ray diffraction. Not only this, the wavelength of the electron was also found to be identical with the calculated by de-Broglie with the help of this eqn
In this way, the dual nature of the electron and quantitative nature of de-Broglie's equation was established.

HEISENBERG'S UNCERTAINTY PRINCIPLE:—

According to Classical mechanics, the Position and Momentum of a moving electron can be determined with great accuracy. When an electron is consider as a wave then it is not possible to know the exact location of the electron on the wave, It was Pointed out by HEISENBERG's in his Uncertainty Principle.

" It is impossible to determine exactly both the Position and the momentum (or velocity) of an electron or of any other moving Particle at the Same time. It means that when an electron behaves as a Particle. its Position Can be detm more or less accuracy/exactly but at the Same time there would be uncertainty about its momentum or Velocity.

Similarly if the Velocity or momentum Can be determined Precisely, there would be uncertainty about its Position.

The uncertainty arises from the fact that when a measurment is Carried Out, the electron under investigation is to be viewed with a Sensitive instrument Such as a microscope, and in this process the light Particle interact with the electron and alter its motion (i.e Velocity). It is not Possible, therefore to Say about the Velocity of electron.

Mathematical interpetation:—

If ΔP is uncertainty in the determination of momentum. and Δx be the uncertainty in the determination of Position then Heisenberg's equation may be written as.

$$\Delta P \times \Delta x \geq \frac{h}{2\pi}$$

Heisenberg's Equation may be stated as follows:—

The product of uncertainty in the Simultaneous determination of the Position and momentum of a Particle is equal to greater than the Plank's Constant.

Statement of Heisenberg's in Case of Energy and Time.

Some times instead of measuring Position and momentum of the System, its energy E, and the time t for which it remains in that energy state are measured, In these case, the uncertainty in measuring energy and time is given by

$$\Delta E \times \Delta T \geq \frac{h}{2\pi}$$, where 'h' is Plank Constant.

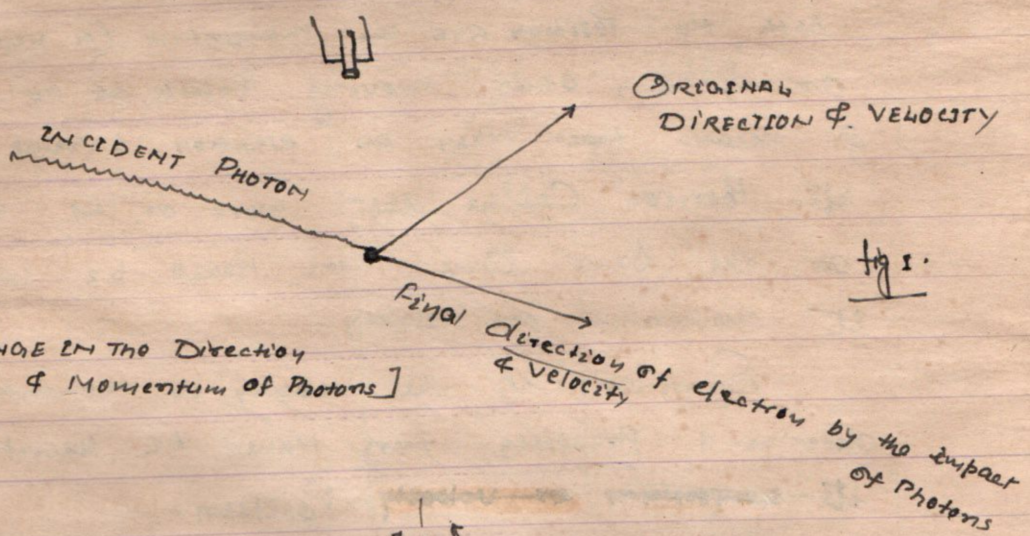


fig 1.

[CHANGE IN THE Direction & Momentum of Photons]